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A discussion of the Cox & Shaw factor for oblique incidence and the film-to-film factor in multiple-film exposures.

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Cox & Shaw's discussion (1930) of the effects produced by an X-ray beam incident obliquely on a photographic film is open to two objections: (*a*) an erroneous expression is used for the transmission by the emulsion, and (*b*) the non-linearity of the curve of density against X-ray exposure is ignored. The consequences of this are discussed below, and an expression is also proposed for use with visually estimated intensities.

(*a*) For incidence at an angle ψ, Cox & Shaw use the form $\exp[-\mu t \sec\psi]$ for the transmission by the film base, and the form $(1-C\sec\psi)$ for the photosensitive layer. The latter is correct only for a single layer of photosensitive grains with $\psi < \sec^{-1} s/d_g$, where s is the average distance between the grains, which have a mean diameter d_g. In this case, the photographic action is proportional to $\sec\psi$. For multiple layers, however, the lower layers are partially shielded, the shielding being proportional to $\sec\psi$, whence the law $\exp[-A\sec\psi]$ readily follows for the transmission. For the important range, as the number of layers of grains increases, the law of transmission alters from $(1-C\sec\psi)$ to

$$\exp[-A\sec\psi] = (\exp[-A])^{\sec\psi} = (1-C)^{\sec\psi}$$
$$\text{with}\quad (1-C) = \exp[-A]\,,$$

cf. Whittaker (1953), who has pointed out this error in Cox & Shaw's derivation. Assuming a linear density–exposure relationship, he has obtained the corrected factor for integrated densities on double-coated film as

$$(D_2)_{\text{corr.}} = \frac{\text{Integrated density at normal incidence}}{\text{Integrated density at angle } \psi}$$
$$= \frac{(1-\exp[-A])(1+\exp[-(A+B)])}{(1-\exp[-A\sec\psi])(1+\exp[-(A+B)\sec\psi])}\,,\dagger \qquad (1)$$

where $B = \mu t$.

(*b*) Experimentally, one measures the equivalent integrated X-ray intensities by comparison with calibration spots, whose densities, D, (not integrated densities) are related to the X-ray intensities, X, by

$$D = D_m(1-\exp[-\alpha X])\,,\ddagger \qquad (2)$$

whence

$$\alpha X = -\log(1-D/D_m)$$
$$= D/D_m + \tfrac{1}{2}(D/D_m)^2 + \tfrac{1}{3}(D/D_m)^3 + \ldots\,. \qquad (3)$$

First assume that the spots have a uniform density. If D_0 and D_ψ are the densities of the calibration spots used for the estimates at angles zero and ψ respectively, we get

$$\frac{1-(D_2)_{\text{corr.}}}{1-(D_2)_{\text{expt.}}} = \frac{(D_\psi - D_0)/D_\psi}{(X_\psi - X_0)/X_\psi}$$
$$= \frac{1+\tfrac{1}{2}(D_\psi/D_m)+\tfrac{1}{3}(D_\psi/D_m)^2+\ldots}{1+\tfrac{1}{2}(D_\psi+D_0)/D_m+\tfrac{1}{3}(D_\psi^2+D_\psi D_0+D_0^2)/D_m^2+\ldots}\,.$$

On putting $\tfrac{1}{2}(D_\psi+D_0) = \bar{D}$, this expression becomes

$$\simeq [(D_m-\bar{D})/D_\psi]\log[D_m/(D_m-D_\psi)] \qquad (4)$$

$$\simeq 1-\tfrac{1}{2}(\bar{D}/D_m)[2-1/\sqrt{(\bar{D}_2)}+\tfrac{1}{3}(\bar{D}/D_m)/\sqrt{(\bar{D}_2)}]\,, \qquad (4a)$$

where $\bar{D}_2$ is the mean of $(D_2)_{\text{corr.}}$ and $(D_2)_{\text{expt.}}$, and therefore the quantity in the square brackets in (4*a*) is approximately unity. Thus the values of $(D_2)_{\text{expt.}}$ will be less than $(D_2)_{\text{corr.}}$, and, for $\bar{D}/D_m \sim \tfrac{1}{3}$, will approximate to the original D_2 given by Cox & Shaw, cf. Fig. (1).

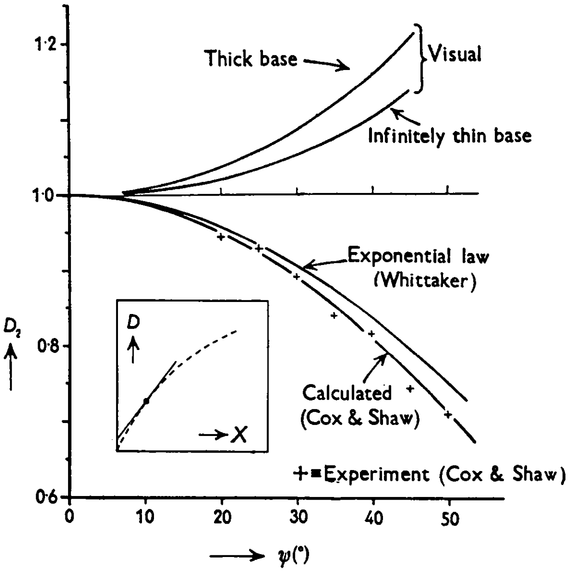

Fig. 1.

It is seen that the good agreement between Cox & Shaw's theoretical curve and their experimental points is largely due to the mutual cancellation of the two errors in their theory. The inset in the figure shows the approximate position of $\bar{D}$ on the curve of equation (2) to fit their experimental points. It is desirable to keep $\bar{D}/D_m$ somewhat lower in order to reduce the error due to this cause.

Since the error is approximately proportional to $\bar{D}/D_m$, the only modification needed in the above treatment to take account of non-uniform spots is to define D_0 and D_ψ as the corresponding mean photographic densities, i.e. as $\int D_0 da \big/ \int da$, etc.

(*c*) We now consider the important case of visual

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† $(D_2)_{\text{corr.}}$ is given above as the reciprocal of Whittaker's result. This has been done to conform to Cox & Shaw's original definition of D_2 as the factor by which intensity data for oblique incidence are to be multiplied.

‡ D_m is evidently the maximum density producible on the film being used.

estimation, where peak densities are compared. For a single layer of photosensitive grains, the total photographic action is proportional to sec ψ, but is now spread over an area that increases in the same ratio, and therefore the action per unit area is independent of ψ, so that $D_2 \equiv 1$. For a multiplicity of layers, the screening of the lower layers must be taken into account, and

$$(D_2)_{\text{visual}} = \left(\frac{\text{Action}}{\text{Area}}\right)_0 \Big/ \left(\frac{\text{Action}}{\text{Area}}\right)_\psi > 1 \,,$$

in contrast to the factor for integrated intensities. The thickness of the emulsion will be negligible compared with the linear dimensions of the spots, and if we assume that for the film base, $t \tan \psi$ is very much less than d, the spot diameter, it follows that the area of the spot is simply proportional to sec ψ, and from equation (1),

$$\begin{aligned} &(D_2)^{t=0}_{\text{visual}} \\ &= \frac{\sec\psi(1-\exp[-A])}{1-\exp[-A\sec\psi]} \times \frac{1+\exp[-(A+B)]}{1+\exp[-(A+B)\sec\psi]} \qquad (5) \\ &= 1+(\sec\psi-1)[(\tfrac{1}{2}A+\varphi)+(\sec\psi-1)\varphi(\varphi-\tfrac{1}{2}B)- \\ &\qquad \tfrac{1}{12}A^2(2-\sec\psi)+\ldots]\,, \end{aligned}$$

where

$$\begin{aligned} \varphi &= \frac{(A+B)\exp[-(A+B)]}{1+\exp[-(A+B)]} \\ &= 0{\cdot}35(A+B)\pm 0{\cdot}02 \quad \text{for} \quad A+B < \tfrac{2}{3}\,, \\ &= 0{\cdot}25\pm 0{\cdot}03 \quad \text{for} \quad \tfrac{2}{3} < A+B < 2{\cdot}3\,. \end{aligned}$$

The second and third terms inside the square brackets together produce an error of less than 0·02 in D_2. To this accuracy, we obtain

$$(D_2)^{t=0}_{\text{visual}} = 1+(\sec\psi-1)(\tfrac{1}{2}A+\varphi)\,. \qquad (5a)$$

We can also estimate the change in formula (5) when the condition $t\tan\psi \ll d$ does not hold. Clearly, the image in the back layer of emulsion is displaced through a distance $t \tan \psi$ with respect to the stronger image in the front layer. As a reasonable estimate of the density distribution for a spot, we take the Gaussian form, $p \exp[-10r^2/d^2]$, so that the density falls to $\frac{1}{12}$ of the peak value at a distance $\frac{1}{2}d$ from the peak. It is not difficult to show that the resultant peak density for the composite spot is approximately

$$\begin{aligned} &p(1+\exp[-(A+B)\sec\psi]) \\ &\times \exp\Bigg[-\frac{10t^2\tan^2\psi}{d^2\sec^2\psi} \\ &\qquad \times \left\{\frac{\exp[-\frac{1}{2}(A+B)\sec\psi]}{1+\exp\left[-(A+B)\sec\psi\left(1-\dfrac{5t^2\tan^2\psi}{d^2\sec^2\psi}\right)\right]}\right\}^2\Bigg] \\ &\simeq p(1+\exp[-(A+B)\sec\psi]) \\ &\qquad \times \exp[-10(t^2/5d^2)\sin^2\psi] \quad (A+B < \tfrac{3}{2})\,, \end{aligned}$$

so that

$$\begin{aligned} (D_2)^{t}_{\text{visual}} &\simeq \exp[10(t^2/5d^2)\sin^2\psi](D_2)^{t=0}_{\text{visual}} \qquad (6) \\ &\simeq \exp[10(t^2/5d^2)\sin^2\psi](1+(\sec\psi-1)(\tfrac{1}{2}A+\varphi))\,. \end{aligned}$$

With typical values of $t/d = \frac{1}{4}$, $A = \frac{1}{3}$, $\varphi = \frac{1}{4}$, the first factor is less important than the second. Typical curves for $(D_2)^{t}_{\text{visual}}$ and $(D_2)^{t=0}_{\text{visual}}$ are plotted in Fig. 1 for comparison with the other curves. The correction factors for visual work are seen to deviate very markedly from those for integrated densities. The effect of a non-linear density–exposure relationship will still be given by equation (4) if $(D_2)_{\text{corr.}}$ is replaced by $(D_2)_{\text{visual}}$; this will make the experimental $(D_2)_{\text{visual}}$ greater still.

Film-to-film factor for multiple-film exposures

Invariant over the foregoing mathematical manipulations is the following property of the absorption:

With the exponential law, the intensity factor from one film to the next in a multiple-film stack at normal incidence is

$$K_0 = \exp[2A+B]\,,$$

and therefore, for incidence at an angle ψ, it is

$$K_\psi = \exp[(2A+B)\sec\psi] = (K_0)^{\sec\psi}\,, \qquad (7)$$

which is a useful result since K_0 is known accurately. The effect of the progressively decreasing radius for successive films is small and produces a negligible error in formula (7).

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The unit-cell dimensions and space groups of some tetrazole compounds.

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The investigation of tetrazole compounds in this laboratory has made available a large number of tetrazole derivatives. Some of these substances which formed good crystals have been investigated by X-ray single-crystal methods with the view of finding one or more suitable for a complete structure determination. The desirability of obtaining direct proof of the existence of the tetrazole ring in these compounds and of the dimensions of this ring is obvious. Table 1 lists the results from six compounds examined. The most satisfactory crystals were all derivatives of 5-aminotetrazole. These compounds were obtained from Drs Ronald A. Henry, William G. Finne-